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ABSTRACT

Analysis of covariance is examined for its usefulness in attribute-by-treatment interaction (ATI) research, where subjects are randomly assigned to treatments but not to levels of the attribute factor. Contrary to what might be expected, the use of a covariate does not confound the interpretation of the interaction between attribute and treatment, because the expected value of the interaction effect is independent of the particular covariate chosen. Independence occurs regardless of whether the attribute variable is discrete or continuous, and regardless of whether the covariate is measured with perfect or less than perfect reliability. The primary implication of these results is that analysis of covariance provides a much needed means for researchers to achieve powerful designs in assessing attribute-by-treatment interactions. (Author/GDC)

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The Use of Analysis of Covariance in Tests of Attribute-by-Treatment Interactions

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Abstract

The use of analysis of covariance is examined for attribute-by-treatment interaction (ATI) research, where subjects are randomly assigned to treatments but not to levels of the attribute factor. Contrary to what might be expected, the use of a covariate does not confound the interpretation of the interaction between attribute and treatment, because the expected value of the interaction effect is independent of the particular covariate chosen. Independence occurs regardless of whether the attribute variable is discrete or continuous, and regardless of whether the covariate is measured with perfect or less than perfect reliability. The primary implication of these results is that analysis of covariance provides a much needed means for researchers to achieve powerful designs in assessing attribute-by-treatment interactions.

The Use of Analysis of Covariance in Tests of Attribute-by-Treatment Interactions

The objective of the current paper is to examine the appropriateness of using covariates in designs which simultaneously investigate the effects of subject attribute factors and experimentally assigned treatments. This issue must be considered in the context of several recent papers (e.g., Evans & Anastasio, 1968; Maxwell & Cramer, 1975; Meehl, 1970) which have cautioned against the indiscriminate use of analysis of covariance (ANCOVA) in single-factor designs where the covariate is correlated in the population with the levels of the factor under investigation. Such situations frequently arise in non-experimental or ex post facto research where subjects are not randomly assigned to groups.

For example, if one's interest is in determining the relationship between an attribute such as sex of subject and the subject's rate of learning in a verbal learning task, it might be reasonable to expect that measures of an ability such as verbal fluency which can predict performance in the experimental task would have a non-zero correlation in the population with sex of subject. As Meehl (1970) and others have pointed out, when one attempts to adjust for differences in verbal fluency, it is unclear whether the adjustment in one's estimate of the relationship between sex and task performance has been correct or too large or too small. It may be the case that, in the population to which a generalization is to be made, the different sexes have different levels of verbal fluency or other abilities and that adjusting for these will inappropriately reduce the apparent magnitude of the relationship between sex and task performance that is of interest. In short,

one's estimate of the effect of variation in sex will take on different expected values depending on the particular ability being used as a covariate. The problem here is essentially the same as that which arises in multiple regression analysis with correlated predictors (Kerlinger & Pedhazur, 1973, p. 93ff.) or in non-orthogonal analysis of variance (Appelbaum & Cramer, 1974). That is, when one's predictors or factors are correlated, the estimate of the extent of relationship between any predictor variable or factor and the dependent variable depends on the other predictors or factors in the model.

One of the major classes of research involving attribute variables has as its main focus the investigation of possible interactions of these variables with randomly assigned experimental treatments (e.g., Cronbach & Snow, 1977). Examples of such attribute-by-treatment interaction (ATI) problems include, among many others that could be cited, differences in what teaching style is most effective (encouraging independence or encouraging conformity) for students of different achievement orientations (via independence or via conformity) (Domino, 1971) as well as interactions of adult cognitive abilities with type of elaboration instructions administered in a verbal learning task (Delaney, in press; DiVesta & Sunshine, 1974).

The seemingly natural conclusion is that the use of ANCOVA in tests of such ATI's is necessarily accompanied by all the same problems of interpretation that beset its use in single-factor non-experimental designs. The current paper attempts to demonstrate that such a conclusion, while apparently reasonable, is totally false. The correct conclusion instead is that the role of ANCOVA in the test of the interaction in

ATI research is identical to that which it plays in purely experimental designs, i.e., in designs where subjects are randomly assigned to conditions. This will be demonstrated by considering the problem first at an intuitive level and then at a more mathematical level. A numerical example will also be cited.

An intuitive argument can be made for why the use of ANCOVA in the interaction test in ATI research must necessarily be similar to its use in the test of the experimental (treatment) factor rather than to its use in the test of the non-experimental (attribute) factor. Consider first that the estimate of the interaction may be arrived at by computing the difference between the estimates of the simple main effects of the experimental factor. The estimate of the simple effect of the experimental factor for, say, high ability subjects is like the estimate obtained in any single-factor ANCOVA in which there is random assignment to levels of the factor. That is, the expectation is that the estimate of the magnitude of this simple effect is not altered by use of a covariate since the population means of the covariate are the same for all treatment groups involved in the simple effect. The same of course is true of the simple test of the treatment effect at the other level(s) of ability. Thus, the use of ANCOVA in assessing the simple effects of the treatment does not cause problems of interpretation. In addition, because the estimate of the interaction is arrived at by simply computing the difference(s) between unbiased estimates of simple effects, the estimate of the interaction is therefore also unbiased.

Note that in any particular sample the numerical estimate of the interaction effect may change slightly in going from analysis of variance

(ANOVA) to ANCOVA since we expect the covariate to have slightly different means in the different treatment conditions because of random sampling. However, in the long run, the expected value of the estimate of the magnitude of the interaction will be identical in the two methods of analysis.

This argument will now be developed in somewhat more rigorous fashion. Consider a two-factor, A x B design, where factor A is an attribute variable which does not permit random assignment to groups but where factor B is an experimental variable. One may assume that each factor has only two levels without loss of generality, since in large designs the overall interaction is simply the composite of that observed in 2 x 2 components of the design (Namboodiri, Carter, & Blalock, 1975, p. 117ff.). Denote the observed cell means on the covariate and dependent variable $\bar{X}_{ij}$ and $\bar{Y}_{ij}$, respectively, where $i = 1, 2$, refers to the level of factor A and $j = 1,2$, refers to the level of factor B. Assuming that the scores on the covariate are determined prior to the experimental manipulation, because of the random assignment to levels of factor B,

$$E(\bar{X}_{11}) = E(\bar{X}_{12}) = E(\bar{X}_{1.}) \tag{1}$$

and

$$E(\bar{X}_{21}) = E(\bar{X}_{22}) = E(\bar{X}_{2.}) \tag{2}$$

Conversely, because subjects were not randomly assigned to levels of factor A, we <u>cannot</u> assert that $E(\bar{X}_{1.}) = E(\bar{X}_{2.})$. Instead it would be more reasonable to expect a non-zero correlation in the population between the level of factor A and an individual's X score.

In the standard ANOVA approach, the concern is estimating and testing the significance of the parameters in the linear model

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + e_{ijk} \tag{3}$$

where μ is the population grand mean, α_i, β_j, and $\alpha\beta_{ij}$ are the typical effect parameters associated respectively with the main effect of factor A, the main effect of factor B and the interaction of factors A and B, and e_{ijk} is the random error associated with the k^{th} individual in cell ij. Because the effect parameters are constrained to sum to zero, in the 2 x 2 design we need only to know α_1, β_1 and $\alpha\beta_{11}$. In an equal-n design, the estimate of the effect of factor B is

$$\hat{\beta}_1 = \bar{Y}_{\cdot 1} - \bar{Y}_{\cdot\cdot} = \tfrac{1}{4}(\bar{Y}_{11} + \bar{Y}_{21} - \bar{Y}_{12} - \bar{Y}_{22}), \tag{4}$$

which has expected value

$$E(\hat{\beta}_1) = \tfrac{1}{4}(\mu_{11} + \mu_{21} - \mu_{12} - \mu_{22}). \tag{5}$$

If a covariate X is used, the adjusted cell means are of the form

$$\bar{Y}'_{ij} = \bar{Y}_{ij} - b\bar{X}_{ij} + b\bar{X}_{\cdot\cdot}, \tag{6}$$

where b represents the common within-cell regression coefficient of Y on X. Thus, since the terms involving the grand mean of the covariate, $\bar{X}_{\cdot\cdot}$, cancel out in the estimate of the effect of factor B, in ANCOVA we have

$$\begin{aligned}\hat{\beta}'_1 &= \tfrac{1}{4}[(\bar{Y}_{11} - b\bar{X}_{11} + \bar{Y}_{21} - b\bar{X}_{21}) - (\bar{Y}_{12} - b\bar{X}_{12} + \bar{Y}_{22} - b\bar{X}_{22})] \\ &= \tfrac{1}{4}[(\bar{Y}_{11} + \bar{Y}_{21} - \bar{Y}_{12} - \bar{Y}_{22}) - b(\bar{X}_{11} - \bar{X}_{12}) - b(\bar{X}_{21} - \bar{X}_{22})].\end{aligned} \tag{7}$$

However, given the independence of b and the differences in the group means, $\bar{X}_{i_1} - \bar{X}_{i_2}$, (Kirk, 1973, p. 464), we have

$$E[b(\bar{X}_{i_1} - \bar{X}_{i_2})] = E(b)E(\bar{X}_{i_1} - \bar{X}_{i_2}). \tag{8}$$

Thus, the expectation of the B effect in ANCOVA is also

$$E(\hat{\beta}'_1) = \tfrac{1}{4}(\mu_{11} + \mu_{21} - \mu_{12} - \mu_{22}), \tag{9}$$

because of the equalities stated in Equations 1 and 2. Note this implies that $E(\hat{\beta}'_1)$ is unaffected by the particular covariate decided upon for a given design or even by the decision not to use any covariate, although

of course these decisions will influence the magnitude of the error term for the analysis. Thus, we see that the expected value of the estimate of the effect of the experimental factor is the same in ANCOVA as in ANOVA. This is of course why there are no problems of interpretation caused by using a covariate to analyze the effects of a factor when one is able to randomly assign subjects to groups.

In the case of factor A, the individual difference factor, we have using ANOVA,

$$E(\hat{\alpha}_1) = \tfrac{1}{4}E(\bar{Y}_{11} + \bar{Y}_{12} - \bar{Y}_{21} - \bar{Y}_{22}) \tag{10}$$

$$= \tfrac{1}{4}(\mu_{11} + \mu_{12} - \mu_{21} - \mu_{22})$$

whereas, using ANCOVA, we have

$$E(\hat{\alpha}_1') = \tfrac{1}{4}[E(\bar{Y}_{11} + \bar{Y}_{12} - \bar{Y}_{21} - \bar{Y}_{22}) - bE(\bar{X}_{11} - \bar{X}_{21}) - bE(\bar{X}_{12} - \bar{X}_{22})]. \tag{11}$$

Because of our inability to randomly assign subjects to levels of factor A we cannot necessarily expect $E(\bar{X}_{11} - \bar{X}_{21})$ or $E(\bar{X}_{12} - \bar{X}_{22})$ to equal zero. Thus, the estimate of the effect of factor A will depend on the particular covariate chosen, as this will determine the extent to which the expectations involving X will depart from zero. This is essentially the statistical basis for the concern over the use of ANCOVA in non-experimental or ex post facto designs (cf. Evans & Anastasio, 1968; Meehl, 1970).

The critical effect in ATI research is naturally the interaction. Similar to the test of factor B, in ANOVA we have

$$\hat{\alpha\beta}_{11} = \tfrac{1}{4}(\bar{Y}_{11} + \bar{Y}_{22} - \bar{Y}_{12} - \bar{Y}_{21}), \tag{12}$$

and

$$E(\hat{\alpha\beta}_{11}) = \tfrac{1}{4}(\mu_{11} - \mu_{12} - \mu_{21} + \mu_{22}). \tag{13}$$

However, in ANCOVA we have

$$\hat{\alpha\beta}'_{11} = \tfrac{1}{4}[(\bar{Y}_{11} - b\bar{X}_{11} + \bar{Y}_{22} - b\bar{X}_{22}) - (\bar{Y}_{12} - b\bar{X}_{12} + \bar{Y}_{21} - b\bar{X}_{21})] \tag{14}$$

$$= \tfrac{1}{4}[(\bar{Y}_{11} - \bar{Y}_{12} - \bar{Y}_{21} + \bar{Y}_{22}) - b(\bar{X}_{11} - \bar{X}_{12}) + b(\bar{X}_{21} - \bar{X}_{22})],$$

which as desired has expected value

$$E(\hat{\alpha\beta}'_{11}) = \tfrac{1}{4}[E(\bar{Y}_{11} - \bar{Y}_{12} - \bar{Y}_{21} + \bar{Y}_{22}) - bE(\bar{X}_{11} - \bar{X}_{12}) + bE(\bar{X}_{21} - \bar{X}_{22})] \tag{15}$$

$$= \tfrac{1}{4}(\mu_{11} - \mu_{12} - \mu_{21} + \mu_{22}).$$

As in the case of factor B, because of random assignment of subjects to treatment groups, the terms involving the cell means of the covariate have dropped out of the final equation. Therefore, we see that the expected value of the estimate of the interaction effect is the same whether or not the covariate is employed.

It should be noted that this conclusion regarding the estimate of the interaction effect is not influenced by the presence of error of measurement in the covariate. It is the case that unreliability of the covariate will affect the value of the common within-cell regression coefficient, b (Kahneman, 1965). Thus, the expected value of any parameter estimate will be affected if that estimate depends on b. We have seen that in ANCOVA the expected value of the attribute effect does depend upon b (see Equation 11), but that this is not the case for the treatment effect nor for the attribute-by-treatment interaction (see Equations 9 and 15). Unreliability of the covariate has no effect on the values of $E(\bar{Y}_{ij})$ or $E(\bar{X}_{ij})$, but these are the only values that determine the expected treatment effect and expected ATI effect.

The preceding discussion has assumed that the attribute or individual difference variable was discrete. However, as Kerlinger and Pedhazur (1973, p. 239ff.) and Cronbach and Snow (1977, p. 26ff.) have suggested, it will frequently be more desirable to treat the attribute variable as a continuous variable. If this is done, it is more convenient to conceptualize the data analysis within a multiple regression framework rather than a standard ANOVA approach. However, the thrust of the preceding discussion concerning the independence of the estimate of the ATI and the covariate applies equally well to the multiple regression situation.

To demonstrate this, consider again a design in which subjects are randomly assigned to one of two experimental treatments. Define the regression model

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + e \qquad (16)$$

where X_1 is the score on the attribute variable in deviation score form, X_2 equals 1 for subjects in treatment 1 and -1 for subjects in treatment 2, X_3 is the term representing the ATI and for each individual is the product of that person's scores on X_1 and X_2, X_4 is the covariate and e is random error. We will begin our discussion of the multiple regression model by considering the six simple correlations among the four predictor variables. These are of interest since it is the case that, if two predictor variables are uncorrelated, the regression coefficient for each will be unaffected by whether or not the other variable is included in the model. Thus, if the correlation, ρ_{ij}, in the population between X_i and X_j is zero, b_i and b_j will also be uncorrelated and in particular b_i will be unaffected if $b_j = 0$ (Draper & Smith, 1966, p. 55, 69ff.).

As before, it is reasonable to assume that the attribute variable X_1 and the covariate X_4 will be correlated, i.e., $\rho_{14} \neq 0$. However, because of the random assignment of subjects to treatment groups, the distribution of X_1 scores will be expected to be the same in the population for both treatment groups, as will the distribution of X_4 scores. Thus

$$E(X_1|X_2 = 1) = E(X_1|X_2 = -1) = E(X_1) \tag{17}$$

and

$$E(X_4|X_2 = 1) = E(X_4|X_2 = -1) = E(X_4). \tag{18}$$

Hence

$$\rho_{12} = \rho_{24} = 0. \tag{19}$$

The remaining three correlations all involve the interaction variable X_3 and can all be shown to be zero in the population. Because X_1 is in deviation score form the expectations of Equation 17 will equal 0, and since for group 1 $X_3 = X_1$ and for group 2 $X_3 = -X_1$ we have

$$E(X_3|X_2 = 1) = E(X_1|X_2 = 1) = 0 = -E(X_1|X_2 = -1) = E(X_3|X_2 = -1). \tag{20}$$

Thus, $\rho_{23} = 0$.

Because the population of X_1 scores is the same for each level of X_2, and because the X_3 score for an individual is the product of the individual's X_1 and X_2 scores where half the X_2 are +1 and half are -1, the expectation is that half of the X_1 scores having a particular value, x_1, will be associated with an X_3 score of $-x_1$ and half will be associated with an X_3 score of $+x_1$. In short,

$$E(X_3|X_1 = x_1) = 0 \text{ for all } x_1, \tag{21}$$

and thus $\rho_{13} = 0$.

Because the population distribution of X_4, like that of X_1, is the same for each level of X_2, the expectation is that half of the X_4 scores have a particular value, x_4, will be associated with some X_3 score say, x_3, and that the other half will be associated with an X_3 score of $-x_3$, i.e.,

$$E(X_3 | x_4) = 0 \text{ for all } x_4. \tag{22}$$

This gives us the desired result of $\rho_{34} = 0$. Thus, the regression coefficient b_3 reflecting the interaction will have the same expected value regardless of whether or not the covariate X_4 is included in the model.

In conclusion, we present two simple numerical examples. First, consider the data shown in Table 1 as an example of the typical case where the attribute variable is viewed as discrete. The data have been arranged according to a 2 x 2 design, where we will assume random assignment was followed for factor B but not for factor A.

Insert Table 1 about here

Note that in the current idealized example the covariate allows perfect prediction within cells and that the covariate is correlated with levels of factor A but not levels of factor B. (A more realistic but less clear example would be generated by adding random error to the Y scores.) The cell and marginal means for the covariate, the original Y scores and the adjusted Y scores are readily seen to be those shown in Table 2.

Insert Tables 2 and 3 about here

From these means we obtain the following estimates of effects:

$$\text{ANOVA: } \hat{\alpha}_1 = 0 \qquad \hat{\beta}_1 = 1 \qquad \hat{\alpha\beta}_{11} = 2 \tag{23}$$

$$\text{ANCOVA: } \hat{\alpha}'_1 = -2 \qquad \hat{\beta}'_1 = 1 \qquad \hat{\alpha\beta}'_{11} = 2 \tag{24}$$

Note the only altered estimate of an effect is that for the non-experimental factor. (In a more realistic example, $\hat{\beta}'_1$ and $\hat{\alpha\beta}'_{11}$ would typically both be slightly changed in going from an ANOVA to an ANCOVA since one would anticipate the means of the covariate would be slightly different at the different levels of B because of sampling error. Also, in behavioral science research one would certainly not expect to remove all within-cell error, but such a contrived example with $\underline{b} = 1$ makes it clear what the adjusted means and effects should be.)

Next, consider the data shown in Table 3 as an example of an ATI design where the attribute factor is treated as a continuous variable. Note that the scores on the dependent variable and on the covariate (designated X_4 in Table 3) are the same as in the previous example. Variables X_1, X_2 and X_3 in Table 3 represent the coding corresponding to the A, B, and AB effects, respectively, where, as before, we assume subjects were randomly assigned to levels of B but not to levels of A. Variable X_1 gives the deviation score form of the subject's score on attribute A. These scores represent an idealized example of the deviation scores that might be obtained in an "extreme groups" design in which only subjects from the extremes of the attribute distribution are selected for participation. X_2 gives the standard effects coding for factor B (Kerlinger & Pedhazur, 1973, p. 121ff.) so that the associated regression weight, b_2, corresponds to $\hat{\beta}_1$ of the previous example. X_3 is simply the product of X_1 and X_2. As has been the case throughout the current paper, the covariate is correlated with the levels of the

attribute factor; here, the sample correlation between X_1 and X_4, r_{14}, is .938. All of the other five correlations between pairs of predictor variables are zero. The regression equations obtained when X_4 is first excluded from and then included in the model are respectively

$$\hat{Y} = 5.000 + .024\ X_1 + 1.000\ X_2 + .439\ X_3, \tag{25}$$

and

$$\hat{Y} = -1.429 - .571\ X_1 + 1.000\ X_2 + .439\ X_3 + 1.286\ X_4. \tag{26}$$

As in the ANOVA example, the parameter estimate corresponding to the attribute factor is altered (substantially) by the use of the covariate. However, the aspect of the two equations of principal interest concerns the values of b_2 and b_3. As anticipated by the above theoretical treatment of the continuous case, the parameter estimates corresponding to the treatment effect and the ATI are not affected in any way by the use of the covariate. (In "real world" applications, b_2 and b_3 would likely be changed slightly by use of X_4 as a predictor, since even though in the population $\rho_{24} = \rho_{34} = 0$ we would anticipate r_{24} and r_{34} would not be exactly zero because of sampling.)

In sum, the crucial point is that one should approach practical situations with the knowledge that the expected value of the estimate of the ATI is unaffected by the use of the covariate.

The scientific importance of this result is substantial for ATI researchers. Because of the independence condition pointed out above, researchers in an area where very sensitive designs are needed (Bracht, 1970) may employ ANCOVA in assessing the effect of primary interest to them (viz. the interaction) without introducing the problems of interpretation which might have been suspected on the basis of recent discussions of the use of ANCOVA in single-factor attribute research. To use Sprott's

terms, the use of ANCOVA in tests of ATI's is valid in that it uses "the information in the X_{ij} to remove from the error variance of the Y_{ij} that portion which is due to the correlation of the X_{ij} without altering the treatment effect" (Sprott, 1970, p. 304).

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Table 1

Artificial Data for an ATI Design

Using a Discrete Attribute Variable

		B	
		1	2
A	1	$X_{111} = 8, Y_{111} = 9$ $X_{112} = 6, Y_{112} = 7$	$X_{121} = 8, Y_{121} = 3$ $X_{122} = 6, Y_{122} = 1$
	2	$X_{211} = 4, Y_{211} = 5$ $X_{212} = 2, Y_{212} = 3$	$X_{221} = 4, Y_{221} = 7$ $X_{222} = 2, Y_{222} = 5$

Table 2

Means for Data of Table 1

		B		
		1	2	
A	1	$\bar{X}_{11}=7$, $\bar{Y}_{11}=8$, $\bar{Y}'_{11}=6$	$\bar{X}_{12}=7$, $\bar{Y}_{12}=2$, $\bar{Y}'_{12}=0$	$\bar{X}_{1\cdot}=7$, $\bar{Y}_{1\cdot}=5$, $\bar{Y}'_{1\cdot}=3$
	2	$\bar{X}_{21}=3$, $\bar{Y}_{21}=4$, $\bar{Y}'_{21}=6$	$\bar{X}_{22}=3$, $\bar{Y}_{22}=6$, $\bar{Y}'_{22}=8$	$\bar{X}_{2\cdot}=3$, $\bar{Y}_{2\cdot}=5$, $\bar{Y}'_{2\cdot}=7$
		$\bar{X}_{\cdot 1}=5$, $\bar{Y}_{\cdot 1}=6$, $\bar{Y}'_{\cdot 1}=6$	$\bar{X}_{\cdot 2}=5$, $\bar{Y}_{\cdot 2}=4$, $\bar{Y}'_{\cdot 2}=4$	$\bar{X}_{\cdot\cdot}=5$, $\bar{Y}_{\cdot\cdot}=5$, $\bar{Y}'_{\cdot\cdot}=5$

Table 3

Artificial Data for an ATI Design

using a Continuous Attribute Variable

A	B	X_1	X_2	X_3	X_4	Y
12	1	5	1	5	8	9
11	1	4	1	4	6	7
12	2	5	-1	-5	8	3
11	2	4	-1	-4	6	1
3	1	-4	1	-4	4	5
2	1	-5	1	-5	2	3
3	2	-4	-1	4	4	7
2	2	-5	-1	5	2	5